

Quark mass density- and temperature- dependent model for strange quark matter

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Abstract

It is found that the radius of a stable strangelet decreases as the temperature increases in a quark mass density-dependent model. To overcome this difficulty, we extend this model to a quark mass density- and temperature- dependent model in which the vacuum energy density at zero baryon density limit B depends on temperature. An *ansatz* which reads $B = B_0[1 - a(T/T_c) + b(T/T_c)^2]$ is introduced and the regions for the best choice of the parameters are studied.

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It has been expected that a new state of matter, namely, the quark-gluon plasma (QGP) can be formed in the relativistic heavy-ion collision. Many signatures of the formation of QGP, such as J/Ψ suppression, strangeness enhancement, thermal dilepton and electromagnetic radiation etc. have been suggested [1]. A lot of them have been found in recent experiments as pointed by CERN collaboration. However whether QGP has been got is still an open question, because many signatures suggested can also be explained by other treatments [2]. To search an unambiguous signature is an essential task for the study of QGP. Due to this reason, the strangelets, the strange quark matter (SQM) in finite lumps, have attracted much attention in recent years. Greiner and his coworkers [3] argued that strangelets might be produced in ultrarelativistic heavy-ion collisions and could serve as an unambiguous signature for the formation of QGP.

Since the speculation of Witten [4] that the strange quark matter (SQM) would be more stable than the normal nuclear matter and nuclei, many models including MIT bag model [5–7], quark-meson coupling model [8], quark mass-density dependent model (QMDD) [9–15], Friedberg-Lee model [16], chiral SU(3) quark model [17] and etc. [18] have been employed to predict the behavior of SQM in the bulk. It is of interest to investigate whether above models which are successful for bulky strange matter can be used to describe strangelets.

This paper evolves from an attempt to study this problem for the QMDD model. This model can provide a dynamical description of the confinement mechanism and explain the stability of SQM successfully via the suggestion of a density dependent masses for u, d and s quarks. It is found that a difficulty emerges if the QMDD model is used to describe strangelets at finite temperature. The radius of strangelet decreases as the temperature increases. This is of course unreasonable. Since the mass of hadrons is observed to be dependent on temperature, we extend the QMDD model to a quark mass density- and temperature-dependent (QMDDT) model, and show that after a suitable choice of the adjusted parameters in the function of $B(T)$, the radius of the strangelet increases with the rise of temperature.

According to the QMDD model, the masses of u, d quarks and strange quarks (and the corresponding anti-quarks) are given by [9,10]

$$m_q = \frac{B}{3\tilde{n}_B}, \quad (q = u, d, \bar{u}, \bar{d}), \quad (1)$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B}{3\tilde{n}_B}, \quad (2)$$

where \tilde{n}_B is the baryon number density, m_{s0} is the current mass of the strange quark and B is the vacuum energy density inside the bag.

The thermodynamic potential is

$$\Omega = \sum_i \Omega_i = - \sum_i T \int_0^\infty dk \frac{dN_i}{dk} \ln \left(1 + e^{-\beta(\varepsilon_i(k) - \mu_i)} \right), \quad (3)$$

where i stands for u, d, s (or $\bar{u}, \bar{d}, \bar{s}$) and the electron $e(e^+)$, μ_i is the corresponding chemical potential. Since the masses of quarks depend on density, the thermodynamic potential Ω is not only a function of temperature, volume and chemical potential, but also of density. How to treat the thermodynamics with density-dependent particle masses self-consistently

is a serious problem and has made many wrangles in the references [10,11,13,15]. Here after we follow the treatment of ref. [13]. The number density \tilde{n}_i , the total pressure p and the total energy density ε are given by

$$\tilde{n}_i = -\frac{1}{V} \left. \frac{\partial \Omega}{\partial \mu_i} \right|_{T, \tilde{n}_B}, \quad (4)$$

$$p = -\frac{1}{V} \left. \frac{\partial(\Omega/\tilde{n}_B)}{\partial(1/\tilde{n}_B)} \right|_{T, \mu_i} = -\frac{\Omega}{V} + \frac{\tilde{n}_B}{V} \left. \frac{\partial \Omega}{\partial \tilde{n}_B} \right|_{T, \mu_i}, \quad (5)$$

$$\varepsilon = \frac{\Omega}{V} + \sum_i \mu_i \tilde{n}_i - \frac{T}{V} \left. \frac{\partial \Omega}{\partial T} \right|_{\mu_i, \tilde{n}_B}, \quad (6)$$

respectively. The baryon number density \tilde{n}_B reads

$$\tilde{n}_B = \frac{1}{3}(\tilde{n}_u + \tilde{n}_d + \tilde{n}_s). \quad (7)$$

The conditions of charge neutrality and chemical equilibrium are

$$2\tilde{n}_u = \tilde{n}_d + \tilde{n}_s + 3\tilde{n}_e, \quad (8)$$

$$\mu_s = \mu_d, \quad \mu_s = \mu_u + \mu_e. \quad (9)$$

Now we are in a position to use the above formulation to investigate the strangelets. Instead of one of plane wave, the density of states $\frac{dN_i}{dk}$ for a sphere with radius R is needed in our calculations. It was given by a multi-reflection theory [19]

$$\frac{dN_i}{dk} = g_i \left[\frac{k^2 V}{2\pi^2} + f_s^{(i)} \left(\frac{m_i}{k} \right) kS + f_c^{(i)} \left(\frac{m_i}{k} \right) C + \dots \right], \quad (10)$$

where $V = \frac{4}{3}\pi R^3$, $S = 4\pi R^2$, $C = 8\pi R$, $g_i = 6$ for quarks and antiquarks, $g_i = 2$ for e and e^+ . The second term on the right hand side of Eq.(10) corresponds to the surface contribution. It is shown [20]

$$f_s^{(i)} \left(\frac{m_i}{k} \right) = -\frac{1}{8\pi} \left(1 - \frac{2}{\pi} \arctan \frac{k}{m_i} \right). \quad (11)$$

This term is zero for massless quarks. The third term on the right hand side of Eq.(10) comes from curvature of the bag surface. It can not be obtained by the multi-reflection theory directly except for two limiting cases $m_i \rightarrow 0$ and $m_i \rightarrow \infty$. Madsen proposed [21]

$$f_c^{(i)} \left(\frac{m_i}{k} \right) = \frac{1}{12\pi^2} \left(1 - \frac{3k}{2m_i} \left(\frac{\pi}{2} - \arctan \frac{k}{m_i} \right) \right). \quad (12)$$

This simple form agrees to the above mentioned two values in the corresponding limits.

For the strangelets, the stability condition reads

$$\frac{\partial F}{\partial R} = 0, \quad (13)$$

where F is the total free energy. And it can be given by

$$F = E - TS, \quad (14)$$

where $E = \varepsilon V$ is the total energy,

$$S = \sum_i S_i = - \sum_i \int_0^\infty dk \frac{dN_i}{dk} [n_i \ln n_i + (1 - n_i) \ln(1 - n_i)] \quad (15)$$

is the entropy, n_i is the distributing function of fermions and i stands for u, d, s (or $\bar{u}, \bar{d}, \bar{s}$) quarks and the electron $e(e^+)$. Substituting Eqs.(10), (11), (12) into Eq.(3) and using Eq.(13), we can obtain the stable radius R of the strangelet, which is a function of temperature.

The curves for F per baryon number A vs. R of QMDD model at zero temperature and at $T = 50\text{MeV}$ are shown in Fig.1 by solid line and dashed line, respectively. The values of A, B, m_{s0} are chosen as

$$A = 20, B = 170\text{MeVfm}^{-3}, m_{s0} = 150\text{MeV}. \quad (16)$$

It is clear that the radius for minimum F decreases as temperature increases. The temperature dependence on radius R is displayed in Fig.2, in which it can be seen that $R(T)$ is a monotonously decreasing function. This is of course unreasonable because the bag is expected to expand as the increase of temperature.

To understand this result physically, let us recall what happens for the masses of nucleons and mesons when temperature increases. In fact, the effective masses of nucleons, effective masses and screening masses of mesons, are all dependent on temperature [22–24]. We can sum the tadpole diagrams and the exchange diagrams for nucleons, the vacuum polarization diagrams for mesons by Thermo Field Dynamics and find that the masses of nucleons and mesons all decrease as temperature increases. This result for ρ -meson is in agreement with recent experiments [25]. According to the constituent quark model, the nucleon is constructed by three quarks and the meson by two quarks. This means that to satisfy quark model we must consider the temperature dependence of the quark mass. But this effect has not been taken into account in Eqs.(1) and (2) if B is a constant. Therefore we come to a conclusion that the unphysical result of QMDD model in studying strangelets comes from the assumption that B is temperature independent.

Although it is possible to find the function $B(T)$ from a calculations of the vacuum energy, but in this paper, instead of the vacuum energy calculation, we study this problem more generally by introducing an *ansatz*, namely

$$B = B_0 \left[1 - a \left(\frac{T}{T_c} \right) + b \left(\frac{T}{T_c} \right)^2 \right], \quad (17)$$

where a, b are two adjust parameters, and $T_c = 170\text{MeV}$ is the critical temperature of the quark deconfinement phase transition. The reasons for our choice are: first, the calculation of $B(T)$ are model-dependent and the results given by different model are very different

[26] [27]; secondly, almost all previous calculations are based on MIT bag model, but now we discuss the QMDD model. In fact we can imagine that the Eq.(17) is the temperature expansion of B in the low temperature regions. Since B is zero when $T = T_c$, a condition

$$1 - a + b = 0 \quad (18)$$

is imposed and only one parameter a can be adjusted.

Introducing Eqs.(17) and (18), we extend the QMDD model to a quark mass density- and temperature- model(QMDTD). The results of our model obtained by substituting Eqs.(17) and (18) into Eqs.(1) and (2) are shown in Figs.(1), (3) and (4). The F/A vs. stable R curve for $T = 50\text{MeV}$, $a = 0.65$ given by QMDTD model is shown in Fig.(1) by dot line. We see that the value of stable radius increases from $R_{(T=0)} = 2.27\text{fm}$ to $R_{(T=50\text{MeV})} = 2.31\text{fm}$, but decreases to $R_{(T=50\text{MeV})} = 2.18\text{fm}$ for QMDD model as shown by dashed-line.

The value of parameter a can affect the result significantly. The range of possible a is determined by physical constraints. For example, there are at least two physical constraints: (1), the stable radius R should increase with temperature; (2), the energy per baryon E/A increases with temperature also. To show the importance of first constraint, we plot the $R(T)$ curves for $a = -0.20, 0.20, 0.40, 0.65, 0.80$ in Fig.(3), respectively. We see that $R(T)$ becomes a monotonously increasing function in the regions $0 \leq T \leq 80\text{MeV}$ when $a \geq 0.65$. On the other hand, the E/A vs. T curves for $a = 1.50, 0.90, 0.80, 0.65, 0.20$ are shown in Fig.(4) to investigate the relevance of the second constraint. We find E/A vs. T curves becomes monotonously increasing function in the same temperature region when $a \leq 0.8$. Therefore the best values for parameter a are in the range

$$0.65 \leq a \leq 0.8. \quad (19)$$

In summary, in order to overcome the difficulty related to the reduction of R with T , we suggested a QMDTD model. An ansatz for the temperature dependence of vacuum energy B (Eq.(17)) was introduced. We found that the parameters a must lies in the range $0.65 \leq a \leq 0.8$. Our model can be used to study strangelet, in addition it can also be employed to address systematically the properties of SQM in bulk.

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I. FIGURE CAPTIONS

Fig.1 The total free energy per baryon F/A as a function of radius R , for temperature $T = 0$ (solid line), $T = 50\text{MeV}$ in QMDD model (dashed line) and $T = 50\text{MeV}$, $a = 0.65$ in QMDTD model (dotted line).

Fig.2 The radius R as a function of temperature T for QMDD model.

Fig.3 The radius R as a function of temperature T for QMDTD model with various values for the parameter a as indicated.

Fig.4 The energy per baryon E/A as a function of temperature T for QMDTD model with various values for the parameter a as indicated.